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ABOUT GEOMETRY OF PENTAGONAL DOUBLE-LAYER SUBSTRUCTURES IN UNIDOM SPACE BAR SYSTEM

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ABSTRACT: The idea of the UNIDOM space bar system was formed by present author, investigating step by step geometry of spherical version of Rhombiicosidodecahedron, Dodecahedron, Rhombicubooctahedron and Icosahedron and next, the same, in polyhedral versions Refs 4-13. In the beginning there were published some information about formulae based on spherical trigonometry. Next there were presented possibilities of space arrangement by models built of the elements accessible in Zometool teachers kit produced in USA on U.S.Pat.Re.3375 Refs 1,14. In Beijing during IASS-APCS Symposium there was used first time the name UNIDOM (UNIfied DOMes), for the reason, that from two bars and one node, it is possible to built many various types of space bar structures, including domes with shape based on well known polyhedrons. This time the attention is turned on geometry specially of pentagonal substructures. In the result whole dome can be built from two repeatable bars and one node, only, as double-layer space bar structure, very regular, rigid and cheap in manufacturing and erection. If the number of used typical bars is a little increased, the possibilities of system are much higher.

Key words: UNIDOM, pentagonal, double-layer, substructures, space bar system, repeatable, regular, geometry, various applications

1. INTRODUCTION

The task of sphere division on regular polygons is well known from years, see e.g. Makowski Refs. 2, 3. The present paper gives certain continuation of previous author's theoretical publications on IASS Symposia: the first time in Bangalore 1988 Ref. 4 and the second in Toronto 1992 Ref. 5 and than on the 4th International Conference on Space Structures in Guildford 1993, Ref. 6. Next, step by step were presented particular examples of models performed of the elements accessible in commercial ZOMETOOL kit (U.S.Pat.Re.3375), Refs 1, 14: Singapore 1997 Ref. 7, Delft 2000 Ref. 8, Nagoya 2001 Ref. 9, Montpelier 2004 Ref. 10, Beijing 2006 Ref. 11, LSCE 2006 in Warsaw Ref. 12 and Bangalore 2007 Ref.13.

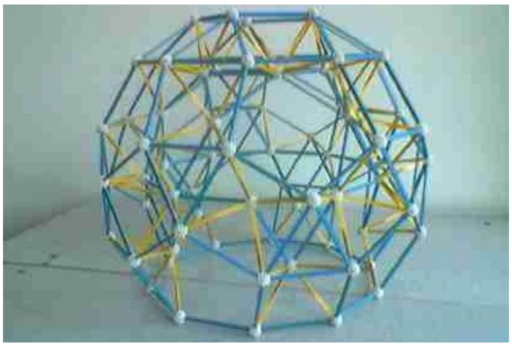

Fig. 1. Example of dome built from basic super-elements with dimensions 1x1 modules (n=1, m=1)

At last, On the IASS-APCS 2006 International Symposium in Beijing Ref. 11 was defined system of prefabricated space bar structures, double-layer, named as UNIfied DOMes. It permits on designing of various bar structures, which can be built of two kinds of bars and one type of node, only (see photographs). By realizations of such objects, limited number of used elements, can permit on application theirs high manufacturing.

By means of two members (blue and yellow – see photos) and one node, only, can be designed many different roofs of large span and others:

- double layer space bar domes with different bar patterns,
- space bar roofs as double layer structures,
- complex buildings with walls and roofs of space bar structure type, structural columns, towers, bridge girders etc.

In the last presentations were shown numerous photographs of the models, of space structures - mainly domes. Now the attention is focused on pentagonal substructures of domes designed in UNIDOM system – single and double-layer type. There, one of more interesting and important thing is geometry of repeatable pentagonal substructures, especially double-layer. Among the others, more important questions were concerning of regularity of connections of substructures in vertexes and edges of neighbouring polygonal double-layer substructures. Now the answer is at all positive.

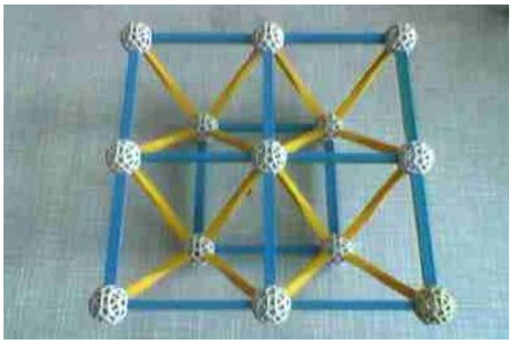

Fig. 2.Basic super-element (n=1, m=1)

The essential advantage of UNIDOM system is possibility to form three dimensional polyhedral space bar domes with double layered structural plane walls. In general, their global shape is based on RHOMBIICOSIDODECAHEDRON shown in the figure 1. or RHOMBICUBOOCTAHEDRON and its modifications. The attention in this paper is focused on the first type polyhedron. Such domes are regular and built of some unified substructures of different size and character. So, their external surface is fulfilled by some regular repeatable plane substructures: rectangular (including square, figure 2), triangular (see figure 3) and pentagonal, see Ref. 13, Tab. 1. As the whole or their suitable parts, they may form large span roofs of different civil engineering objects with different architectonic effects. Moreover, as it was shown in previous papers, applying a few more members related together with golden proportion, there, can be obtained beautiful artistic effects! In the photos, for the reason of limited number of elements, were shown very small models, only.

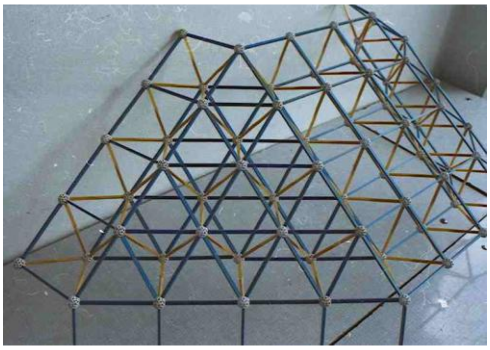

Fig. 3. Connection of triangular and rectangular sub-structures (n=3, m=4)

Table 1. Numbers of elementary walls in particular polyhedrons

Type of Polyhedron	Type of wall			
	Rectangle	Square	Triangle	Pentagon
Rhombiicosidodecahedron	30		20	12
Dodecahedron				12
Icosahedron			20	
Rhombicubooctahedron	12	6	8	
Cube		6		
Octahedron			8	

2. GLOBAL GEOMETRY OF UNIDOM SPACE BAR DOMES

The global outlook of the rhombiicosidodecahedron is shown in the figure 4. In the case, when dimensions of its triangular walls are coming to zero, we obtain dodecahedron, shown in the figure 5.

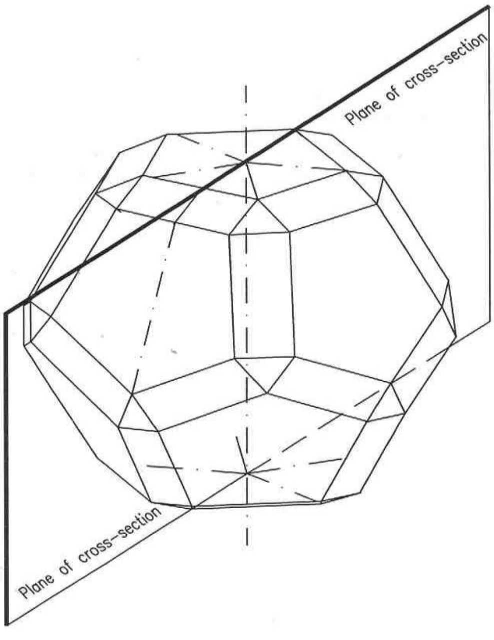

Fig.4. Rhombiicosidodecahedron

For determination of detailed geometry and dimensions of the both basic types of polyhedrons, they are cut by plane symmetry as in the figures 4,5. Here is put very important question, about global dimensions of whole structure, dependently on the dimensions of one rectangular wall a×b. Dimension a – means simultaneously the side of pentagons and b – concern side of triangle.

Moreover, each of the above sections is composed of some essential bars a=n×B, b=m×B, denoted here as **blue** (B). The colours used in this paper are identical as in ZOMETOOL teachers kit.

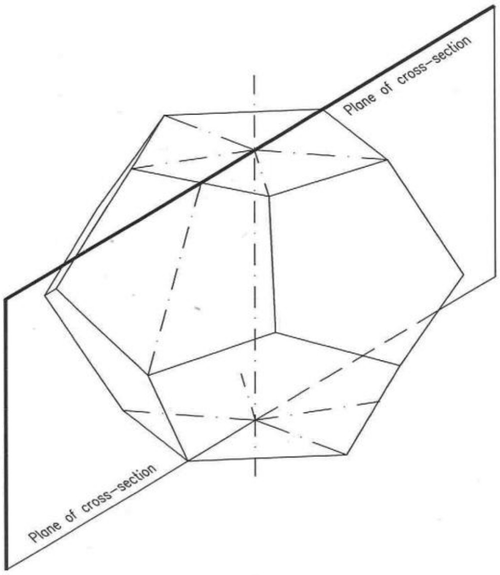

Fig.5. Dodecahedron

Therefore, the fundamental question in each task is: in what a way will change global size of dome, when be applied new dimensions a and b.

The both polyhedrons are inscribed into certain spheres. In the figure 6 external circle concern of rhombiicosidodecahedron, drawn by continuous lines and smaller circle show size of dodecahedron, drawn by dashed lines. There we can do following observations:

- if the both polyhedrons are composed from identical size pentagons, the rhombiicosidodecahedron (R=7.505B) is bigger than dodecahedron (R=5.47688B),
- the vertexes of dodecahedron lies on the same radiuses as centres of gravity of trianglular walls for rhombiicosidodecahedron, (see figure 6 – dash & dot radiuses).

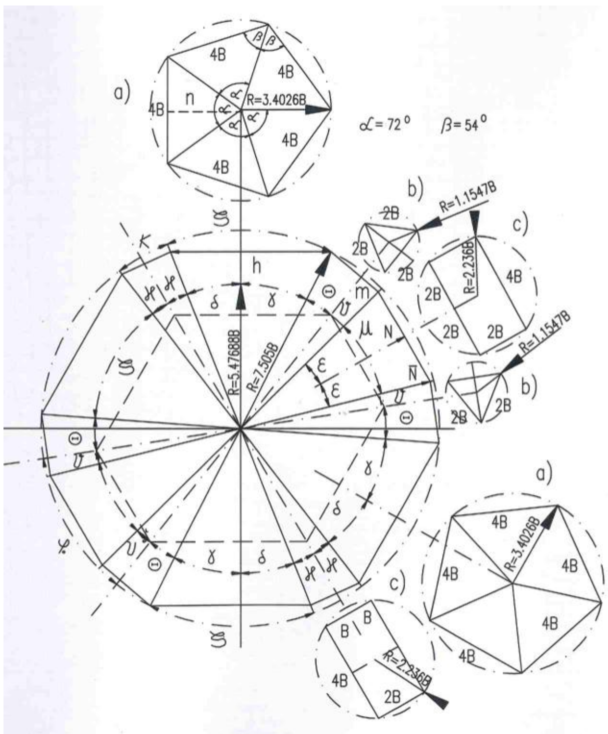

Fig.6. Detailed geometry of rhombiicosidodecahedron

The figure 6 and next, are prepared for dome with essential sizes of substructures: a=4, b=2 (see figures 6a,b,c).

3. ESSENTIAL ANGLES FOR DODECAHEDRON

For determination of radiuses of the spheres into which are inscribed as well rhombiicosidodecahedron as dodecahedron, too, should be analysed properties of particular elementary walls. All its vertexes lies on these spheres.

3.1. Dimensions of pentagon

Position of pentagon on sphere surface is shown in the top of figure 6a (R=3.4026B). There is assumed, that each side of the pentagon has length a=4B. What means, that it is composed of the four **blue** bars. From the figure we can calculate:

$$\sin\gamma = \frac{3.402603233B}{R} \quad ,$$

$$\gamma = \arcsin\left(\frac{3.402603233\,B}{R}\right) \; , \tag{2}$$

$$n = 3.402603233\,B\cos 36^\circ = 2.752763841B \; ,$$

$$tg\delta = \frac{2.752763811\,B}{R\cos\gamma} \; ,$$

$$\cos\gamma = \sqrt{1-\sin^2\gamma} = \frac{1}{R}\sqrt{R^2-(3.402603233B)^2} \; ,$$

$$tg\delta = \frac{2.752763811\,B}{\sqrt{R^2-(3.402603233\,B)^2}} \; ,$$

$$\delta = arctg\left(\frac{2.752763811\,B}{\sqrt{R^2-(3.402603233B)^2}}\right) . \tag{3}$$

$$\zeta = \gamma + \delta \tag{4}$$

3.2. Dimensions of triangle

Position of triangle on sphere surface is shown at the right side in the top of the figure 6b (R=1.1547B). There is assumed, that each side of the triangle has length b=2B what means, that it is composed of the two **blue** bars. From the figure we can calculate as follow.

Radius of the circle in which triangle is inscribed is:

$$\overline{R} = \frac{B}{\cos 30^\circ} = 1.154700538B \; .$$

Moreover:

$$m = 1B\,tg30^\circ = 0.5773502692B \; ,$$

$$\sin\Theta = \frac{1.154700538B}{R} \quad ,$$

$$\sin\upsilon = \frac{0.5773502692B}{R} \quad ,$$

$$\Theta = \arcsin\left(\frac{1.154700538\,B}{R}\right) \quad , \tag{5}$$

$$\upsilon = \arcsin\left(\frac{0.5773502692\,B}{R}\right) \quad , \tag{6}$$

$$\overline{\Theta} = \Theta + \upsilon \quad . \tag{7}$$

3.3. Dimensions of rectangular

Here are considered rectangular (R=2.236B) and two nearest triangles, figures 6b,c,b:

$$B = \frac{b}{2} = R\sin\overline{\kappa} \quad ,$$

$$\overline{N} = R\cos\overline{\kappa} \quad ,$$

$$\mu = \varphi_o - 2\upsilon = \varphi_o - 2\arcsin\left(\frac{0.5773502692\,B}{R}\right) \quad , \tag{8}$$

$$\varepsilon = \frac{\mu}{2} \quad ,$$

$$N = \overline{N}\cos\varepsilon \quad ,$$

$$tg\kappa = \frac{B}{N} = \frac{B}{R\cos\overline{\kappa}\,\cos\varepsilon} \quad ,$$

$$\kappa = arctg\left(\frac{B}{R\cos\overline{\kappa}\,\cos\varepsilon}\right) \quad ,$$

$$\mathrm{K} = 2\kappa = 2arctg\left(\frac{B}{R\cos\overline{\kappa}\,\cos\varepsilon}\right) \quad . \tag{9}$$

4. DETERMINATION OF EXTERNAL RADIUSES FOR DOMES

Before calculation of all particular angles for our polyhedrons, we must to determinate its external radiuses. First should be analysed dodecahedron and angle φ_0.

4.1. External radius and angles for dodecahedron (a=4B)

In the case of dodecahedron: K=0 and $\upsilon = 0$. Therefore,

$$\mu = \varphi_0 = 2\arcsin\frac{2B}{R}$$

and we can form following equation (see figure 6):

$$2\zeta + 2\overline{\Theta} + \mu = \pi \quad . \tag{10}$$

After replacing proper symbols by adequate formulae, when B=1, we obtain:

$$2\arcsin\left(\frac{3.402603233B}{R}\right) + 2arctg\left(\frac{2.752763811B}{\sqrt{R^2-(3.402603233)^2}}\right) + 2\arcsin\left(\frac{2B}{R}\right) = \pi$$

The searched value of radius R fulfilling this equation is equal:

R=5.476 880 4 B .

Next we can calculate (Eqns (2,3,4):

$$\gamma = 38.40869235^o \quad ,$$

$$\delta = 30.17317768^o \quad ,$$

$$\zeta = \gamma + \delta = 68.58187003^o \quad ,$$

and the length of the side of pentagon a=4B produce angle:

$$\varphi_o = 2\arcsin\left(\frac{2B}{R}\right) = 42.83626079^o \, .$$

4.2. External radius and angles for rhombiicosidodecahedron

Here we start from equation (a=4B) see figure 6):

$$4\zeta + 4\overline{\Theta} + 2\mu + 2\mathrm{K} = 2\pi \quad . \tag{10}$$

After replacing the angles by proper formulae, we have:

$$4\arcsin\left(\frac{3.402603233B}{R}\right) + 4arctg\left(\frac{2.752763811B}{\sqrt{R^2-(3.402603233)^2}}\right) +$$

$$+ 4\arcsin\left(\frac{0.5773502692B}{R}\right) + 4\arcsin\left(\frac{1.154700538B}{R}\right) +$$

$$+ 2\left(\varphi_o - 2\arcsin\left(\frac{0.5773502692B}{R}\right)\right)\varphi_0 + 4arctg\left(\frac{2B}{R\cos\varepsilon\sqrt{1-\frac{B^2}{R^2}}}\right) = 2\pi$$

The searched value of radius R fulfilling this equation is now equal:

R=7.505 B .

Next we can calculate (Eqns (2,3,4):

$$\gamma = 26.96063187^o \quad ,$$

$$\delta = 21.51782409^o \quad ,$$

$$\zeta = \gamma + \delta = 48.47845596^o \quad ,$$

and the length of the rectangle a=4B produce angle:

$$\mu = \varphi_o - 2\upsilon = 30.73400322^o \, .$$

Next angles for triangle and rectangle are:

$$\varepsilon = 15.36700161^o \quad ,$$

$$\kappa = 7.937471229^o \quad ,$$

$$\mathrm{K} = 15.87494246^o$$

5. POSSIBLE VARIANTS OF PENTAGONAL SUBSTRUCTURES

Up to the moment were discovered three possible bar patterns for pentagon. The first, plane - irregular and two next, which can be built by means of standard nodes similar as provided in the ZOMETOOL teachers kit. It these two cases all used nodes in whole dome will be identical.

5.1. Plane pentagonal double-layer substructure

In this case, the most natural, the pentagonal substructures, single- or double-layer, are plane. So, whole dome has classical polyhedral shape. There can be applied for example two bar patterns as in the figures 7b,c (are shown in three sectors bars of upper net, cross-braces and bars of lower net). This solution has very inconvenient property – all five triangles are irregular – with sides lengths: 4B + 3.4026B + 3.4026B, figure 7. It generate some new bars with next lengths (here not calculated). This way are needed not only new bars, but next nodes with other bars inclinations.

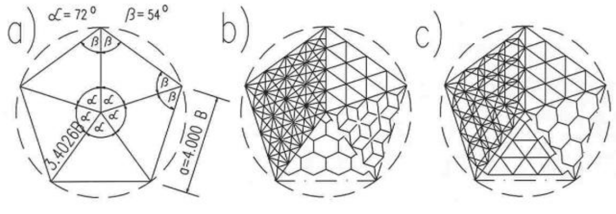

Fig.7. a) geometry of pentagon, b) c) two possible bar patterns

5.2. Concave regular pentagonal double-layer substructure

This solution is from manufacturing point of view the best. All triangular substructures are at all regular with sides length a=4B, figure 8. So it is identical geometry as for all other triangles in whole dome! The week side of this solution is concave character of the pentagon. The depth of this concavity is significant H=2.202B. In this solution all nodes in whole dome are identical.

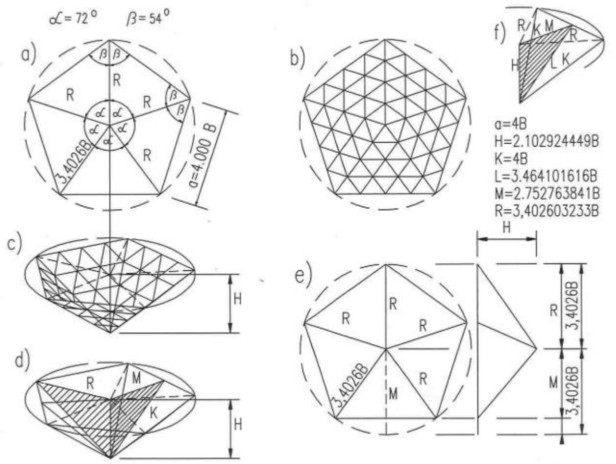

Fig.8

5.3. Convex high pentagonal double-layer substructure (Fig.9)

In this solution all used nodes are identical as in whole dome. There pentagonal substructure is convex with significant elevation H=1.701B.

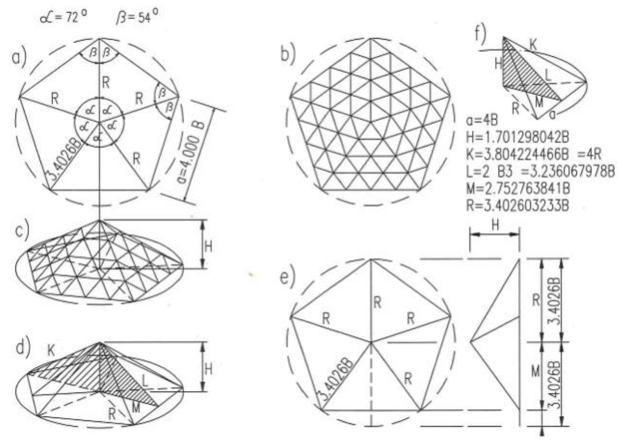

Fig.9

It can be regarded as positive architectural effect. But there we find certain negative property, too. The all inclined bars in triangles belong to **red** bars. So, are needed new members, and bar pattern in triangles is irregular. It explain photo in the figure 10.

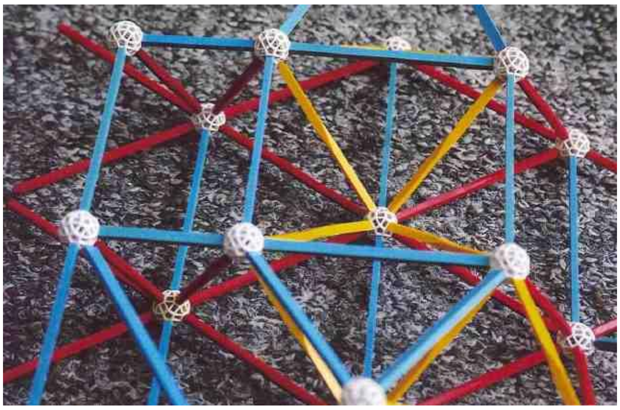

Fig. 10. Visible from down: bottom irregular layer (blue), triangular – irregular mesh (red and blue members) and cross braces – red and yellow members

5.4. Convex low pentagonal double-layer substructure

This solution is shown in the figures 11 and 12. There all nodes are typical as in whole dome, but pentagon has convexity with small H=0.525B and bar pattern is irregular as in the figure 12b. There along five corner radiuses are not continuous bars and this direction is interrupted by smaller pentagons. Moreover in surface of triangles we have squares and rectangulars (with golden proportions). Certain explanation gives here the photo in the figure 13.

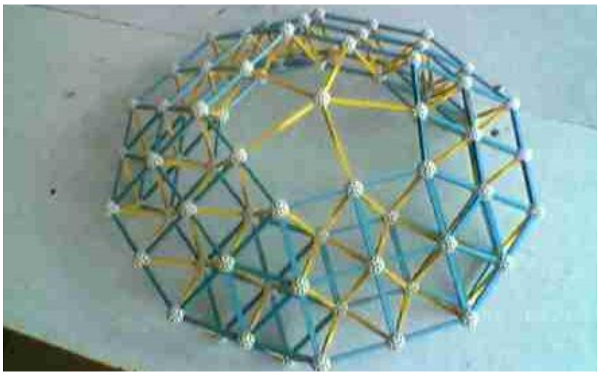

Fig.11

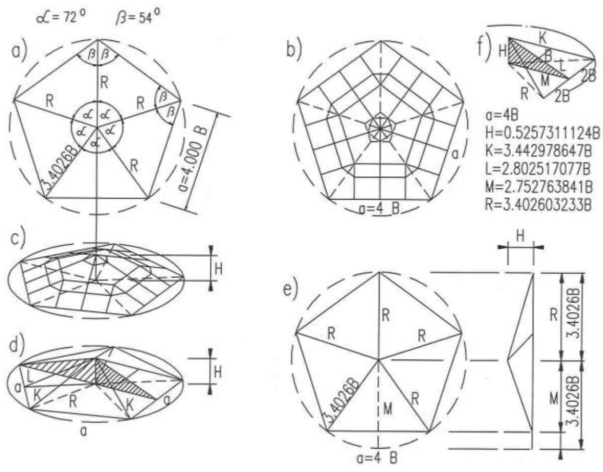

Fig.12

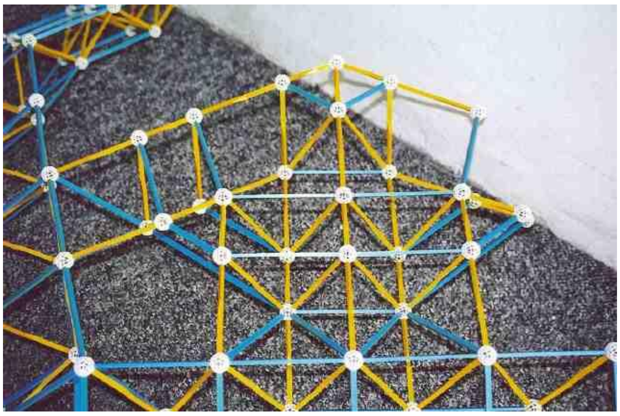

Fig. 13. Bar pattern in the pentagonal substructure of the Fig.12. Visible: not continuous bars on edge between two triangles; irregular pentagon and small triangle

6. OTHER POSSIBLE ARRANGEMENTS OF PENTAGONS

In the paper presented in Singapore, Ref. 7, at 1997, were pointed some next bar patterns for pentagon. They seems to be more irregular and less useful for space bar double-layer domes. All the examples were proved by models.

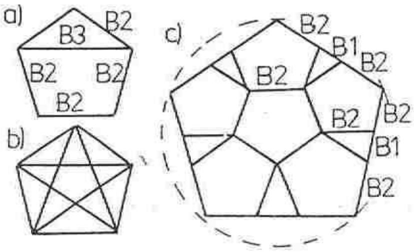

Fig. 14. Possible arrangements of pentagon

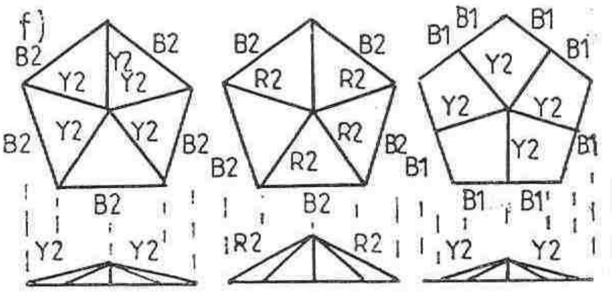

Fig. 15. Convex variants of pentagon covering

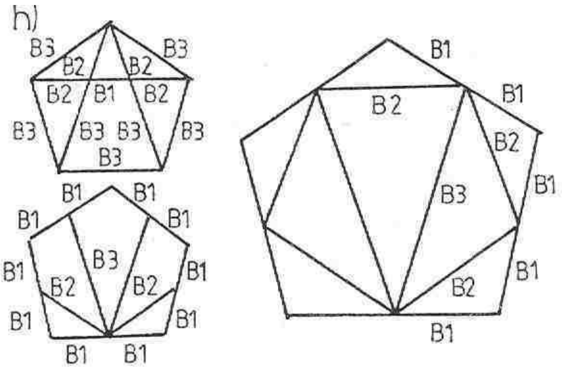

Fig. 16. Possible arrangements of pentagon by ZOMETOOL elements

The attention should be turned especially on the first case shown in the figure 15 and denoted by letter f). There yellow bars are directed to pentagon vertexes. It can give the next version, more regular, of pentagon fulfilment.

The examples given in the figures 14-16 show, that we shell expect much more possible arrangements of pentagon. Some of them can be sufficiently regular.

7. WIDER PHOTOGRAPH DOCUMENTATION

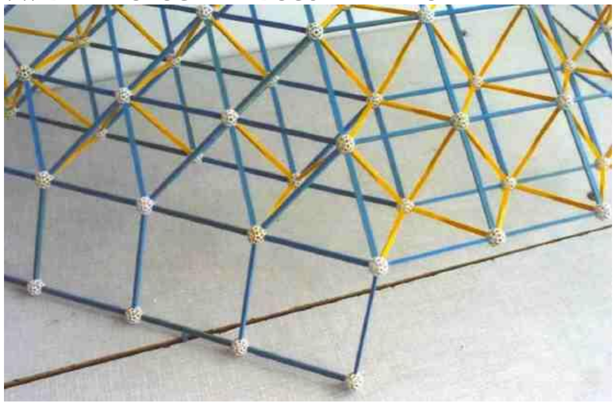

Fig. 17. Example of regular bar mesh along common edge of triangular and rectangular walls

Fig. 18. Possible arrangements of pentagon - the other view on model of the Fig. 13

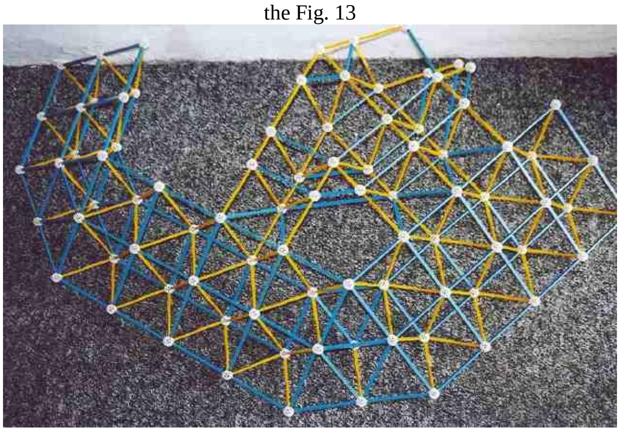

Fig. 19. Possible arrangements of pentagon - the other view on model of the Fig. 13

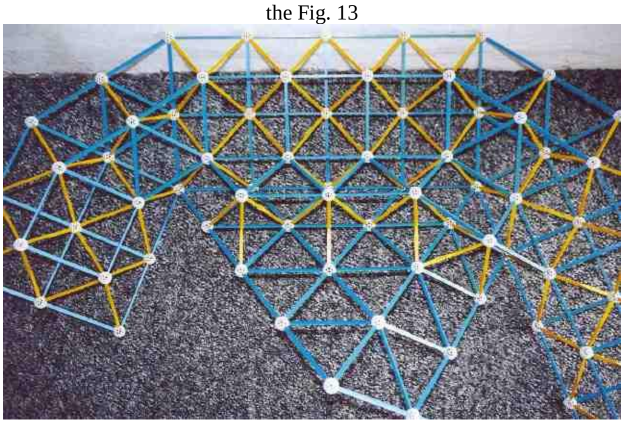

Fig. 20. View from down on: substructures surrounding pentagon (three rectangular and two triangular walls) and on fulfilment on pentagon (in middle, down)

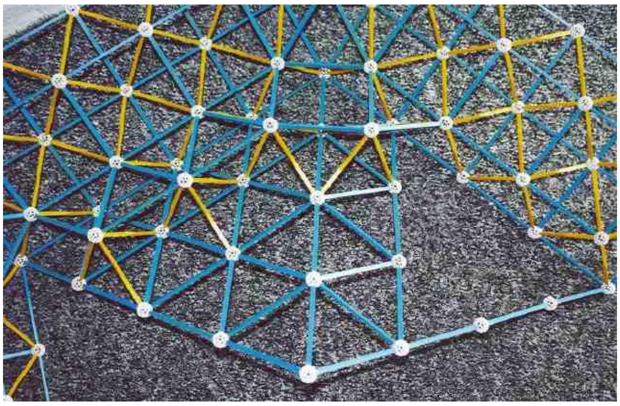

Fig. 21. Example of at all regular bar pattern of: rectangular, triangular and pentagonal walls; view from down (from interior)

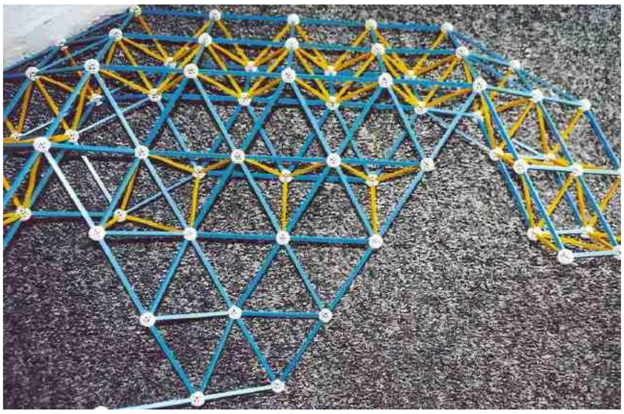

Fig. 22. Example of at all regular bar pattern of: rectangular, triangular and pentagonal walls; view from outside of dome

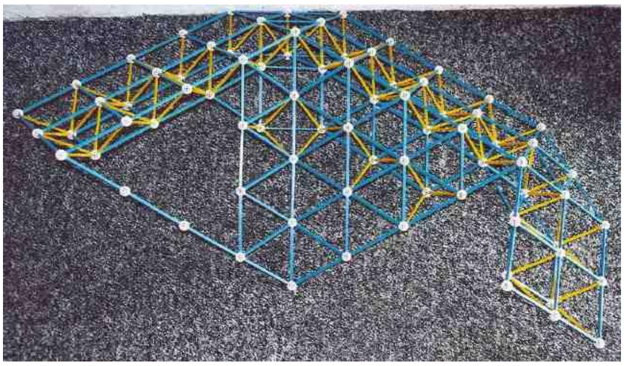

Fig. 23. The other view on models of the Figs 21, 22

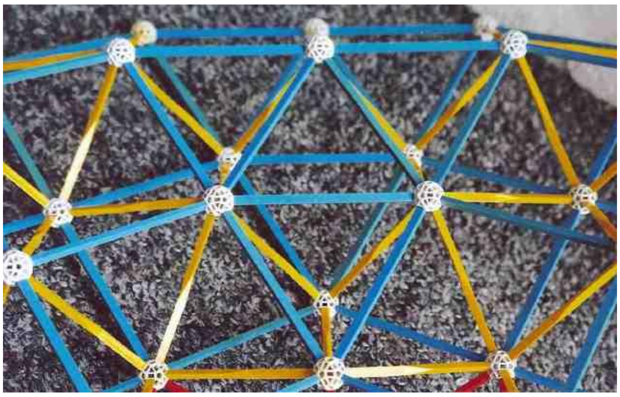

Fig. 24. Closing up on regular connection of triangular and rectangular

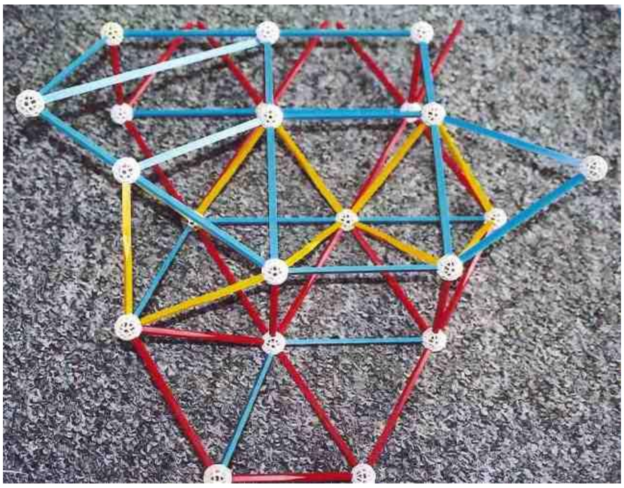

Fig. 25. Closing up on part of the model of the Fig. 10

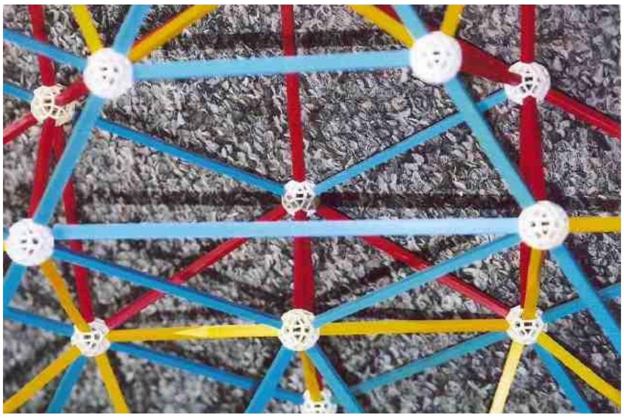

Fig. 26. The other closing up on part of the model of the Fig. 10

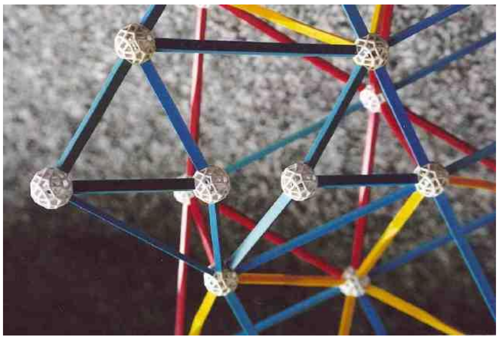

Fig. 27. The other closing up on part of the model of the Fig. 10 – certain proposal of fulfilment of irregular pentagon on edge (not finished)

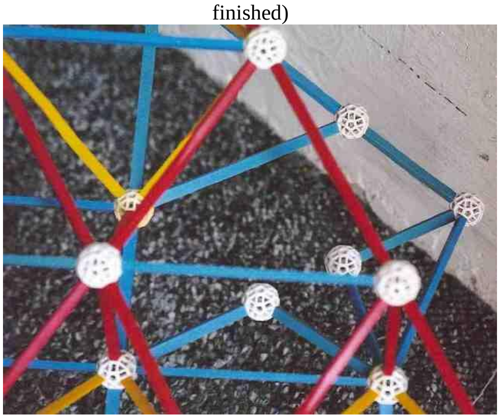

Fig. 28. Closing up on the model of the Fig. 27 – view from down

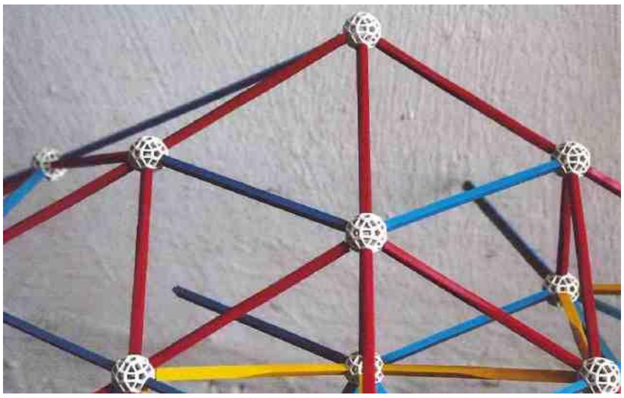

Fig. 29. Detail of mesh for convex variants of pentagon (Fig. 9 and Fig. 15 in the middle). Top part of connection of two triangles.

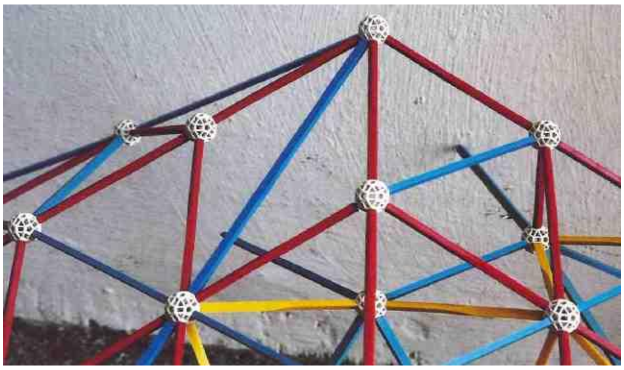

Fig. 30. The other solution for the detail of model of the Fig. 29

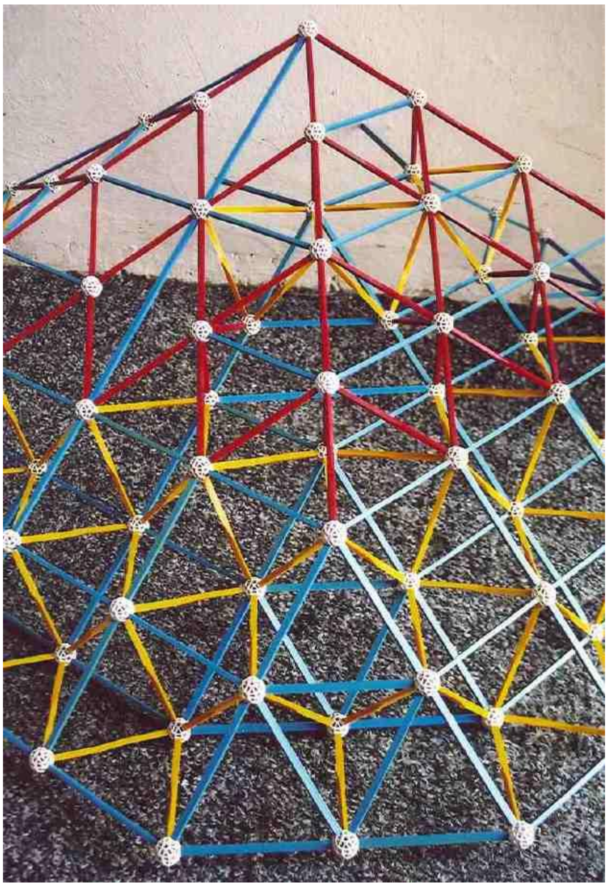

Fig. 31. General view on model of the Figs 29,30

8. FINAL REMARKS AND CONCLUSIONS

The paper in certain sense close principal investigations concerning possibilities of forming various kinds of structures, which can be regarded as belonging to UNIDOM space bar system.

This paper complete information provided especially in the last few publications Refs 9-13, explaining essential questions concerning of geometry of domes based on global shape of rhombiicosidodecahedron and dodecahedron. As the fundamental conclusion of shown this time results is conclusion, that whole such domes can be made as double-layer type and made of three unified elements. Only in the cases shown in the chapter 5.1 are needed new nodes. In the cases of pentagonal substructures from chapters 5.3 appears nee bars (all in the proportions proposed by ZOMETOOL teachers kit – all having golden proportions.

9. REFERENCES

1. S.Baer: Zome primer, elements of zonohedra geometry, two and three dimensional growths of stars with five fold symmetry. Zomeworks Corporation, Albuquerque, New Mexico, 1970.
2. Z.S. Makowski: Räumliche Tragwerke aus Stahl. m.b.H. Düsseldorf, 1963.
3. Z.S. Makowski: Course on the Analysis, Design and Construction of Braced Domes. Space Structures Research Centre, University of Surrey, (app. 1984).
4. J.B. Obrębski: Family of Spherical Domes with Two-Level Repeatability. IASS Symposium on Innovative Applications of Shells and Spatial Forms. 21-25 Nov.1988, Bangalore, India.
5. J.B. Obrębski: On the Conception of the Family of Spherical Domes with Two-Levels of Repeatability. Int. IASS-CSCE Congress 13-17.7.1992, Toronto, Canada.
6. J.B. Obrębski: Non-Conventional Description of Complicated Space Structures. Fourth Int. Conf. on Space Structures, University of Surrey, 6-10 Sept.1993, Guildford, UK.
7. J.B. Obrębski, J. Grzędzielski: On the Space Structures with Golden Proportion of Their Elements, IASS International. Symposium on Shell & Spatial Structures, 10-14.11.1997, Singapore, pp. 661-670.
8. J.B.Obrębski: On Procedures for Shape and Form Finding on Example of Some Families of Space Bar Structures. Int. IASS WG 15th Structural Morphology Conf. Delft, Netherlands, 17-20.08.2000, pp. 123-130.
9. J.B.Obrębski: On Mechanical Behaviour of Space Bar Structures Built of Elements Having Golden Proportions, International IASS Symposium , Nagoya, Japan, 9-14.10.2001, pp. 232-233 +CD-RO6.
10. J.B.Obrębski: Some Examples of Space Structures Morphology Based on Bars Golden Proportions, Intern. IASS Symposium on Shell and Spatial Structures from Models to Realization. Sept. 20-24, & 5th Structural Morphology Seminar Sept. 17-18, 2004, Montpellier, France.
11. J.B. Obrębski, UNIDOM – Proposal of the system for space bar structures. IASS-APCS, 16-19.10.2006 Beijing, pp.86-87.
12. J.B. Obrębski, UNIDOM – Space bar system. XII LSCE, 1.12.2006, pp. 76-87.
13. J.B. Obrębski: More on morphology of UNIDOM space bar system. Structural Engineering World Congress, 2-7.11.2007, Bangalore, India.
14. ZOMETOOL Manual 1.0. Zomeworks Corp. P.O.Box 712 Albuquerque, New Mexico 87103.

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